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Vibrational Control of Underactuated Mechanical Systems: Control Design Through Averaging Analysis

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An open loop vibrational control for an underactuated mechanical system with amplitude and frequency modulation is investigated. Since there is no direct external input to an unactuated joint, the dynamic coupling between the actuated and unactuated joint is utilized for controlling the unactuated joint. Feedback linearization has been performed to fully incorporate the known nonlinearities of the underactuated system considered. The actuated joints are firstly positioned to their desired locations, and then periodic oscillatory inputs are applied to the actuated joints to move the remaining unactuated joints to their target positions. The amplitudes and frequencies of the vibrations introduced are determined through averaging analysis. A systematic way of obtaining an averaged system for the underactuated system via a coordinate transformation is developed. A manipulator in the zero gravity space can be vibrationally controlled in the event of actuator failure. A control design example of the 2R planar manipulator with a free joint with no brake is provided.

Key Words : Averaging, Feedback Linearization, Open Loop Control, Underactuated Manipulator, Vibrational Control

1. Introduction

An underactuated mechanical system refers to the system with fewer number of actuators than the degrees of freedom that the system possesses. Therefore, manipulators with passive or free joints become naturally underactuated systems since the number of control inputs is smaller than the number of generalized coordinates or the dimension of the configuration space. Recently, control of underactuated systems draws great attention to reduce the number of actuators and/or sensors, and to improve the reliability by a fault-tolerant design of fully-actuated manipulators working in hazardous areas or with dangerous materials. It is particularly important to control the failed joints of the space robots working in outer space. Referring that an active joint is the one which is fully controlled via an actuator, and that a passive joint is the one which has no actuation but equipped with a passive element like a brake, and that a free joint is the one which can move freely, the underactuated systems are defined as those with passive and/or free joints.

Control of the unactuated parts of underactuated mechanical systems is in general achieved by utilizing either kinematic or dynamic couplings. Examples of the kinematic coupling are provided by first order nonholonomic systems such as wheeled mobile robots and dextrous robot hands (Murray et al., 1994). The equations of these systems are drift-free with input entering linearly as

$$\dot{x}=\sum_{i=1}^{q} g_i(x)\, u_i$$

where each g_i: $R^p \rightarrow R^p$ is assumed to be a smooth vector field, each input u_i is a piecewise analytic function of time, and $q<p$ is assumed in general, where p and q denote the dimension of

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the vector space considered and the number of inputs, respectively. The second class of systems characterized by dynamic coupling is provided by numerous examples; a crane system, the classical cart-pole system, the Acrobot, and the manipulators with flexible elements. The equations of the second class involves a drift term accounting for gravitational, centripetal, Coriolis, and/or elastic forces with inputs entering affinely as

$$\dot{x} = f(x) + \sum_{i=1}^{q} g_i(x) u_i$$

where the first term $f(x)$: $R^p \rightarrow R^p$ is called a "drift" term because in the absence of control input, the motion drifts in the direction of the vector field f.

The class of underactuated systems considered in this paper belongs to the second class. It is also noted that underactuation does not always imply uncontrollability. The controllability depends on the structure of the system considered. All the examples in the above are controllable. However, in the case of the planar 2R manipulator in section 4 all linearized equations at any operating points are not controllable.

The underactuated system with passive joints has been investigated by several researchers. Arai and Tachi (1991) proved that the number of active joints must be equal or greater than the number of passive ones in order to control the passive ones. They also developed a Cartesian space controller to bring all joints to their desired set points (Arai et al., 1993). Saito et al. (1994) developed a two link underactuated brachiation robot which is capable of moving along crossbars using only one actuator. Bergerman and Xu (1996) investigated a variable structure control for three link manipulator with one passive joint in both joint and Cartesian spaces. One control strategy for underactuated systems with passive joints is to move the passive joints to set points first and lock them with brakes, and then control the remaining active parts. Compared with the works for the systems with passive joints, controls of the systems with free joints are rare.

Recently, scholastic papers which applied periodic oscillations to control the manipulators with free joints have appeared (Suzuki et al., 1996; Suzuki and Nakamura, 1997). Note that the active joint variables appearing in the unactuated joints dynamics can be considered as varying system parameters. Therefore, the periodic movement of an active joint provides a parametric vibrational control to the dynamics of unactuated joints. Suzuki et al. (1996) and Suzuki and Nakamura (1997) investigated an oscillatory control based on Poincare map analysis. De Luca et al. (1997) also investigated a constructive open loop control which involves nilpotent approximation and iterative steps.

Vibrational control is a control technique which utilizes high frequency zero mean vibrations to modify the behavior of dynamical systems in a desired manner. Motivated by the stabilization of an inverted pendulum by a fast vertical oscillation of the support point, the vibrational control theory for linear finite dimensional systems was rigorously introduced in (Meerkov, 1980). The theory for nonlinear systems had been matured in the middle of 1980's by Bellman et al., (1986a, 1986b). The theory has also been extended to the parabolic partial differential equations (Bentsman and Hong, 1991; Bentsman and Hong, 1993) and functional differential equations (Bentsman et al., 1991; Lehman et al., 1994; Lehman and Shujaee, 1994; Shujaee and Lehman, 1997).

In this paper, a prescribed end point steering problem for underactuated systems with unactuated joints via partial feedback linearization and vibrational control is investigated. The control design consists of two stages. The first stage linearizes the system partially, and applies a proper control to drive the active joints to their desired locations. At the end of first stage the positions of unactuated joints will be arbitrary. Then periodic inputs to the active joints are applied to move the remaining free joints to their desired positions via dynamic coupling. Proper magnitudes and frequencies for the oscillatory inputs are determined through averaging analysis.

The contributions of the paper are as follows: The paper is the first investigation of vibrational control to underactuated systems. Averaging analysis is extended to the system with the derivatives and anti-derivatives of vibrations. A systematic way of obtaining averaged systems for underactuated systems is developed. The control strategy in this paper provides a viable tool in the case that the conventional control scheme is not available and/or actuator failure occurs. The class of systems considered in this paper allows a drift term and assumes free joints with no brake. A manipulator in outer space with a failed joint may be vibrationally controlled. The utilization of multiple magnitudes and frequencies of vibrational inputs is proposed.

This paper is divided into four sections. In Section 2, the equations of motion and partial linearization of underactuated systems are formulated. In Section 3, the vibrational control is investigated. Vibrations are introduced to an actuated joint for the purpose of invoking the dynamic coupling. An averaged system is obtained. In Section 4, as an application, a planar 2R manipulator is vibrationally controlled. An averaged system is also demonstrated. Conclusions are given in Section 5.

2. Control Problem

Consider a k degrees of freedom open loop mechanism with joint variables $q_1, \cdots, q_k$. It is assumed that each joint has a single degree of freedom and only $m<k$ joints are active and the remaining $l=k-m$ joints are unactuated. It is assumed that all joint variables, either actuated or unactuated, can be measured.

Using the Lagrange method, one can derive the equations of motion of the system, and rearrange the equations so that the coordinates for actuated joints are grouped in $q_1 \in R^m$ and the coordinates for unactuated joints are grouped in $q_2 \in R^l$. Hence the final form of the equations of motion for an underactuated mechanical system is represented as

$$M_{11}\ddot{q}_{11}+M_{12}\ddot{q}_2+C_1(q,\ \dot{q})+G_1(q)=f \quad (1)$$

$$M_{21}\ddot{q}_1+M_{22}\ddot{q}_2+C_2(q,\ \dot{q})+G_2(q)=0 \quad (2)$$

where the vector functions $C_1(q,\ \dot{q}) \in R^m$ and $C_2(q,\ \dot{q}) \in R^l$ contain Coriolis and centripetal terms, the vector functions $G_1(q) \in R^m$ and $G_2(q) \in R^l$ contain gravitational terms, $f \in R^m$ represents the input generalized force produced by the m actuators at the active joints. Hence, similiar to an ordinary robot, the dynamic equations for an underactuated system can be written as

$$M(q)\ddot{q}+C(q,\ \dot{q})+G(q)=Bf \quad (3)$$

where

$$q=[q_1^T,\ q_2^T],\ M(q)=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$

$$B=\begin{bmatrix} I_{m\times m} \\ 0_{l\times m} \end{bmatrix},\ C=[C_1^T,\ C_2^T]^T\ G=[G_1^T,\ G_2^T]^T$$

Note that M is a symmetric positive definite matrix.

Now consider equation (2). The term M_{22} is an invertible $l\times l$ matrix as a consequence of the uniform positive definiteness of the inertia matrix M in (3). Therefore, we may solve for $\ddot{q}_2$ as

$$\ddot{q}_2=-M_{22}^{-1}(M_{21}\ddot{q}_1+C_2+G_2). \quad (4)$$

Substituting (4) into (1) yields

$$\bar{M}_{11}\ddot{q}_1+\bar{C}_1+\bar{G}_1=f \quad (5)$$

where $\bar{M}_{11}=M_{11}-M_{12}M_{22}^{-1}M_{21}$, $\bar{C}_1=C_1-M_{12}M_{22}^{-1}C_2$, and $\bar{G}_1=G_1-M_{12}M_{22}^{-1}G_2$. A partial feedback linearizing controller can therefore be defined for equation (5) according to

$$f=\bar{M}_{11}u+\bar{C}_1+\bar{G}_1$$

where $u \in R^m$ is an additional control input yet to be defined. Note that the $m\times m$ matrix $\bar{M}_{11}$ is itself symmetric and positive definite. The complete system up to this point may be written as

$$\ddot{q}_1=u \quad (6)$$

$$M_{22}(q)\ddot{q}_2+C_2(q,\ \dot{q})+G_2(q)=-M_{21}(q)u \quad (7)$$

Since the input-output relation from u to q_1 in (6) is linear, the active part q_1-dynamics has been completely linearized. However, considering the full state vector q only partial linearization has been achieved.

3. Vibrational Control and Averaging

Vibrational control is a control technique which utilizes the high frequency zero mean vibrations in the system to modify the behavior of the dynamical system in a desired manner. The parameters in which vibrations are introduced could be either system parameters or control input.

The method of averaging is an asymptotic method which permits the analysis of dynamic behavior of a time-varying system via a time-invariant (averaged) system, which is obtained by averaging of the right hand side of the original time-varying system. In this paper once all the active joints reach their desired set points, periodic inputs are applied to the active joints for the purpose of moving the remaining unactuated joints to their target positions via the dynamic coupling. Since the input is periodic, each active joint returns back to its original position in each period. The design issue now becomes how to move the unactuated joints to their set positions. Since the system becomes time-varying, the input magnitudes and frequencies are calculable through the trajectory analysis of an averaged system.

Define state variables as $x=[q_2, \dot{q}_2]^T \in R^n$ in which $n=2l$. Then the state equation for (7) becomes

$$\dot{x}=\begin{bmatrix} x_2 \\ -M_{22}^{-1}(x_1, q_1)\{C_2(x_1, x_2, q_1, \dot{q}_1) \\ +G_2(x_1, q_1)+M_{21}(x_1, q_1)\ddot{q}_1\} \end{bmatrix}$$

$$\overset{\text{def}}{=} X(x; q_1, \dot{q}_1, \ddot{q}_1) \tag{8a}$$

$$\overset{\text{def}}{=} X(x, \lambda) \tag{8b}$$

where X: $R^n \rightarrow R^n$. In (8a) q_1, $\dot{q}_1$, and $\ddot{q}_1$ are considered as system parameters. In (8b) λ is introduced in order to emphasize one selective parameter in which vibrations are introduced. It is remarked that only a subset of $\{q_1, \dot{q}_1, \ddot{q}_1\}$ may appear in (8a) depending on the structure of the underactuated systems considered. λ is taken as the second highest term among $\{q_1, \dot{q}_1, \ddot{q}_1\}$. For example, if q_1 and $\dot{q}_1$ appear in (8a), then $\lambda=q_1$. Assuming that all q_1, $\dot{q}_1$, $\ddot{q}_1$ exist in (8a) introduce an oscillatory input into (8b) as follows

$$\lambda(t) \overset{\text{def}}{=} \dot{q}_1(t) \rightarrow \lambda_0+\gamma(t),$$

where

$$\gamma(t)=af(\omega t) \tag{9}$$

In (9), λ_0 is a constant, and $\gamma(t)$ is a zero mean periodic function in which a and ω are amplitude and frequency, respectively. The following relations hold

$$\ddot{q}_1=a\omega f'(\omega t), \text{ and } q_1=\frac{a}{\omega}F(wt) \tag{10}$$

where f' and F are the derivative and anti-derivative of f, respectively. Substituting (9) and (10) into (8) yields

$$\dot{x}=X(x, \lambda_0+\gamma(t))$$
$$=\begin{bmatrix} x_2 \\ -M_{22}^{-1}(x_1,\frac{a}{\omega}F(\omega t)\Big\{C_2(x_1,x_2,\frac{a}{\omega}F(\omega t), \\ af(\omega t))+G_2(x_1, \frac{a}{\omega}F(\omega t)) \\ +a\omega M_{21}(x_1, \frac{a}{\omega}F(\omega t))f'(\omega t)\Big\} \end{bmatrix}. \tag{11}$$

It is now assumed that (11) can be decomposed into two parts as follows;

$$\dot{x}=X_0(x, \omega t, \frac{1}{\omega})+\omega X_1(x, wt). \tag{12}$$

The second term consists of the terms which involves the derivatives of $\lambda(t)$. Now, the second term is utilized as the generating equation which transforms (12) into a standard form as follows;

$$\dot{\xi}=X_1(\xi, t). \tag{13}$$

Let $h(t, c)$: $R\times R^n \rightarrow R$ be the general solution of (13) which is T-periodic. Note that $c\in R^n$ can be uniquely defined once initial conditions $\xi(t_0)\in \Omega \subset R$ are provided.

Introduce a new variable $q(t)$ as follows; (the Lyapunov substitution)

$$x(t)=h(wt, q(t)). \tag{14}$$

Differentiating both sides of (14) with respect to t, the following is obtained

$$\dot{q}(t)=\left[\frac{\partial h(\omega t, q(t))}{\partial q}\right]^{-1} X_0(h(\omega t, q(t)), \omega t, \frac{1}{\omega}). \tag{15}$$

In slow time scale such that $\tau=\omega t$ with $z(\tau)=q(t)$ and $\varepsilon=1/\omega$, the following standard form is obtained.

$$\dot{z}(\tau)=\varepsilon\left[\frac{\partial h(\tau, z(\tau))}{\partial z}\right]^{-1} X_0(h(\tau, z(\tau)), \tau, \varepsilon) \quad (16)$$

All the above derivation is now summarized in the following theorem.

Theorem 1: Consider an underactuated system (8) with the assumption (12). Then, there exists a transformation $h: R^+ \times R^n \to R^n$ under which the system is transformed into the following standard form

$$\dot{z}=\varepsilon f(z, t, \varepsilon) \quad (17)$$

where $z \in U \subseteq R^n$, $0<\varepsilon \ll 1$, $f: R^n \times R \times R^+$ is T-periodic. ■

Finally, an averaged system is defined as

$$\dot{y}=\varepsilon \bar{Y}(y) \quad (18)$$

where

$$\bar{Y}(y) \stackrel{\text{def}}{=} \frac{1}{T}\int_0^T f(y, \tau, 0)\, d\tau \quad (19)$$
$$=\frac{1}{T}\int_0^T \left[\frac{\partial h(\tau, y)}{\partial y}\right]^{-1} X_0(h(\tau, y), \tau, 0)\, d\tau$$

By applying the theory of averaging, it is known that there exists $\varepsilon_0>0$ such that for all $0<\varepsilon \le \varepsilon_0$, the hyperbolic stability properties of (17) and (18) are the same.

Theorem 2 (The Averaging Theorem, Guckenheimer and Holmes (1983), p. 167): There exists a C^r, $r \ge 2$, transformation of coordinates $z=y+\varepsilon w(y, t, \varepsilon)$ under which (17) becomes

$$\dot{y}=\varepsilon \bar{f}(y)+\varepsilon^2 f_1(y, t, \varepsilon),$$

where f_1 is of period T in t. Moreover

(i) If $z(t)$ and $y(t)$ are solutions of (17) and (18) based at z_0, y_0, respectively, at $t=0$, and $|z_0-y_0|=O(\varepsilon)$, then $|z(t)-y(t)|=O(\varepsilon)$ on the time scale $t \sim 1/\varepsilon$.

(ii) If p_0 is a hyperbolic fixed point of (18) then there exists $\varepsilon_0>0$ such that, for all $0<\varepsilon \le \varepsilon_0$, (17) possesses a unique hyperbolic periodic orbit $\gamma_\varepsilon(t)=p_0+O(\varepsilon)$ of the same stability type as p_0.

(iii) If $z^s(t) \in W^s(\gamma_\varepsilon)$ is a solution of (17) lying in the stable manifold of the hyperbolic periodic orbit $\gamma_\varepsilon=p_0+O(\varepsilon)$, $y^s \in W^s(p_0)$ is a solution of (18) lying in the stable manifold of the hyperbolic fixed point p_0 and $|z^s(0)-y^s(0)|=O(\varepsilon)$, then $|z^s(t)-y^s(t)|=O(\varepsilon)$ for $t \in [0, \infty)$. Similar results apply to solutions lying in the unstable manifolds on the time interval $t \in (-\infty, 0]$. ■

4. Application

In this section, an example of underactuated mechanical system with free joint; a planar 2R manipulator is introduced. In the case of 2R manipulator linear control theory is not applicable since the linearized system at any operating point is not controllable.

4.1 A planar 2R manipulator

Figure 1 shows a planar 2R manipulator on the horizontal plane. Using the Lagrange equation, the following equations of motion are obtained.

$$M_{11}(\theta_2)\ddot{\theta}_1+M_{12}(\theta_2)\ddot{\theta}_2+C_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2)=\tau$$
$$M_{12}(\theta_2)\ddot{\theta}_1+M_{22}(\theta_2)\ddot{\theta}_2+C_2(\theta_2, \dot{\theta}_1, \dot{\theta}_2)=0$$

where

$$M_{11}(\theta_2)=m_1 l_{1c}^2+m_2 l_1^2+m_2 l_{2c}^2+2m_2 l_{2c} l_1 \cos\theta_2 + I_1+I_2,$$
$$M_{12}(\theta_2)=m_2 l_{2c}^2+m_2 s_2 l_{1c}\cos\theta_2+I_2,$$
$$M_{22}=m_2 l_{2c}^2+I_2,$$
$$C_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2)=-m_2 l_{2c} l_1 \sin\theta_2(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2^2),$$
$$C_2(\theta_2, \dot{\theta}_1, \dot{\theta}_2)=2m_2 l_{2c} l_1 \dot{\theta}_1^2 \sin\theta_2$$

Note that the gravity term does not appear in the equations, and check that the linearized system at an operating point $(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)=(\theta_1, \theta_2)$ is not controllable.

Following the procedure in Sec. 2, the following partially linearized system is obtained

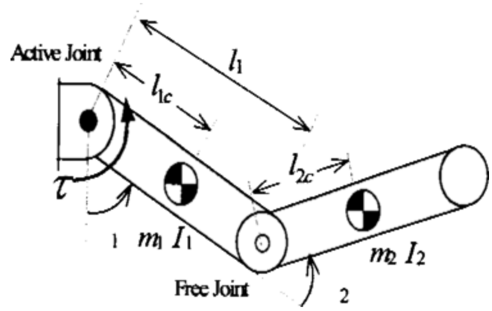


Fig. 1 A planar 2R free-joint manipulator.

$$\ddot{\theta}_1 = u \tag{20a}$$
$$\ddot{\theta}_2 = -(1+n\cos\theta_2)\ddot{\theta}_1 - n(\dot{\theta}_1)^2\sin\theta_2 \tag{20b}$$

where $n = m_2 l_1 l_{2c}/(m_2 l_{2c}^2 + I_2)$ is a constant. Now assume that the active joint θ_1 has been positioned at its desired location with an appropriate control action. For instance,

$$u = \ddot{\theta}_{1d} + k_1(\dot{\theta}_{1d} - \dot{\theta}_1) + k_2(\theta_{1d} - \theta_1)$$

would suffice. Now restricting our control task to the second equation, $\ddot{\theta}_1$ and $\dot{\theta}_1^2$ become varying parameters in the θ_2 dynamics. Define the state variables as $x_1 = \theta_2$, $x_2 = \dot{\theta}_2$. Then the state equation becomes

$$\dot{x}_1 = x_2,\ x_1(0) = x_{10}$$
$$\dot{x}_2 = -(1+n\cos x_1)\ddot{\theta}_1 - n\sin x_1(\dot{\theta}_1)^2,$$
$$x_2(0) = x_{20} \tag{21}$$

Introduce vibrations (9) in Sec. 3 as follows;

$$\lambda(t) = \dot{\theta}_1(t) = \alpha\sin\omega t \tag{22a}$$

Then, the following also holds

$$\theta_1(t) = -\frac{\alpha}{\omega}\cos\omega t,\ \text{and}\ \ddot{\theta}_1(t) = \alpha\omega\cos\omega t \tag{22b}$$

Substituting (22a, b) into (21) yields

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ -n\sin x_1(\alpha\sin wt)^2\end{bmatrix} + w\begin{bmatrix}0\\ -\alpha\cos wt(1+n\cos x_1)\end{bmatrix}. \tag{23}$$

Note that (23) is in the form of (12). Therefore the generating equation of (13) takes the form

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix} = \begin{bmatrix}0\\ -\alpha\cos t(1+n\cos\xi_1)\end{bmatrix}. \tag{24}$$

A general solution of (24) is

$$h(t,c) = \begin{bmatrix}h_1(t,c)\\ h_2(t,c)\end{bmatrix} = \begin{bmatrix}c_1\\ c_2 - \alpha(1+n\cos c_1)\sin t\end{bmatrix}. \tag{25}$$

A coordinate transformation through (25) is defined as

$$\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} = \begin{bmatrix}q_1(t)\\ q_2(t) - \alpha(1+n\cos q_1(t))\sin t\end{bmatrix}. \tag{26}$$

Therefore, under (26) (15) takes the form

$$\dot{q}_1(t) = q_2 - \alpha(1+n\cos q_1)\sin wt,\ q_1(0) = x_{10}$$
$$\dot{q}_2(t) = n\alpha\sin q_1 \sin wt(-q_2 + n\alpha\cos q_1\sin wt),$$
$$q_2(0) = x_{20} \tag{27}$$

In slow time $\tau = wt$ with $q(t) = z(\tau)$, and $\varepsilon = 1/w$, the following standard form is obtained

$$\dot{z}_1(\tau) = \varepsilon[z_2 - \alpha(1+n\cos z_1)\sin\tau],$$
$$z_1(0) = x_{10} \tag{28}$$
$$\dot{z}_2(\tau) = \varepsilon n\alpha\sin z_1\sin\tau(-z_2 + n\alpha\cos z_1\sin\tau),$$
$$z_2(0) = x_{20}$$

Finally, by applying the definition of averaging, Eq. (19), the following averaged system is obtained.

$$\dot{y}_1 = \varepsilon y_2,\ y_1(0) = x_{10} \tag{29}$$
$$\dot{y}_2 = \varepsilon\frac{n^2\alpha^2}{4}\sin 2y_1,\ y_2(0) = x_{20}$$

It is noted that the initial conditions of averaged systems are not in general the same as those of original system.

4.2 Control design

Figure 2 shows trajectories of transformed system (27) with $\alpha = 0.5$ $\omega = 4\pi$ departing from various initial states. Figure 3 shows a phase portrait of (29). Comparing Fig. 2 with Fig. 3, it is observed that the behavior of transformed system (27) is well described by that of the averaged system (29). Figure 4 shows the trajectories of the averaged system with different α's. It is observed that an arbitrary state can be reached by modulating α. It is also noted that changing ω does not influence the averaged behavior. However, by increasing ω the amplitude of $\theta_1(t)$ in (22b) gets smaller and Poincare map gets dense. Therefore, more precise movement can be achieved by increasing ω.

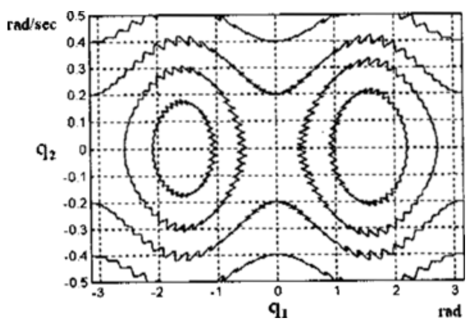


Fig. 2 Trajectories of transformed system (27) with $\alpha = 0.5$, $\omega = 4\pi$.

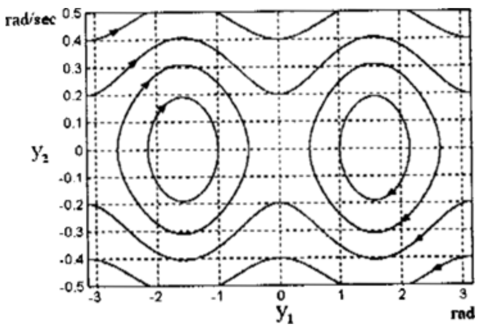


Fig. 3 Phase portrait of averaged system (29) with $\alpha = 0.5$, $\omega = 4\pi$.

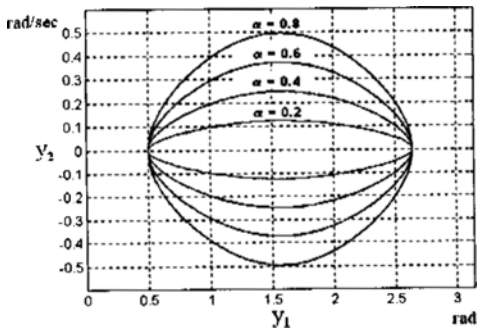
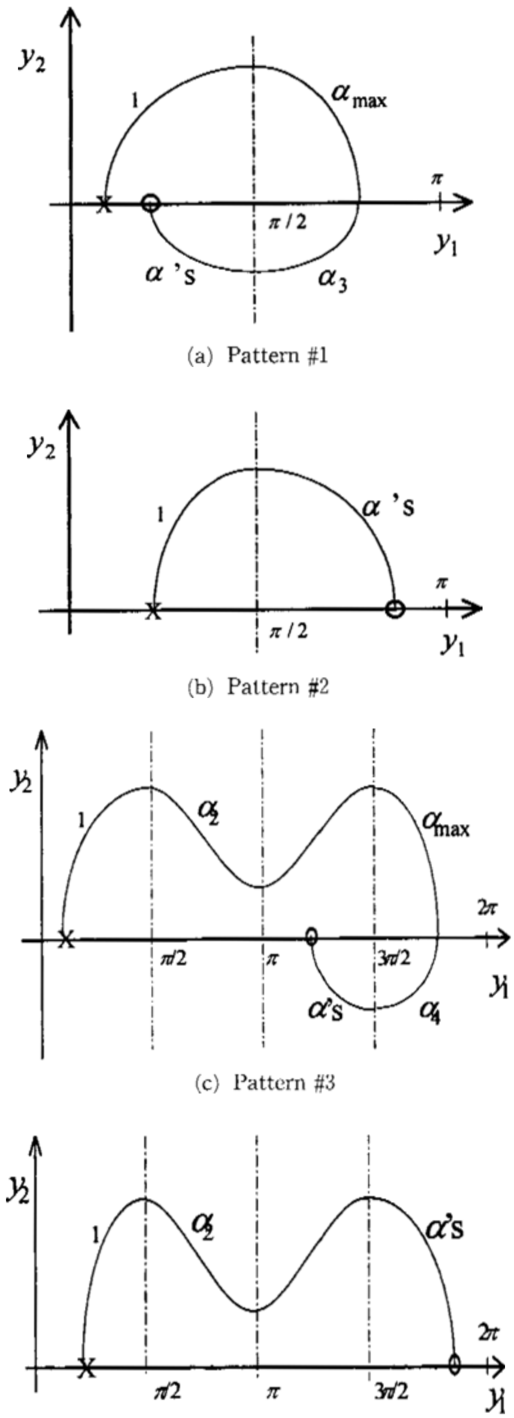


Fig. 4 Trajectories of averaged system (29) with various α's ($\omega = 4\pi$).



(a) Pattern #1

(b) Pattern #2

(c) Pattern #3

(d) Pattern #4

Fig. 5 Control strategies.
(× : Initial state, ○ : Desired state)

Let (y_1, y_2) be a present state and (y_{1d}, y_{2d}) be a desired state. From (29) the following relationship holds

$$\frac{n^2\alpha^2}{2}\cos^2 y_1 + y_2^2 = \frac{n^2\alpha^2}{2}\cos^2 y_{1d} + y_{2d}^2. \quad (30)$$

Therefore, the amplitude of the vibrations to be introduced is calculated as follows:

$$\alpha = \sqrt{\frac{2(y_2^2 - y_{2d}^2)}{n^2(\cos^2 y_{2d} - \cos^2 y_2)}} \quad (31)$$

Due to the flow of system (29) positioning problems may slightly differ depending on initial and target positions. Four control strategies are sketched in Fig. 5. Specifically, consider a positioning problem from the initial state (0.5, 0) to the desired state (1.0, 0) which corresponds to the case of Fig. 5(a). One can provide vibrations with frequency ω and amplitude α_1 to the active joint θ_1. When θ_2 reaches $\pi/2$, the input amplitude in the second quarter is switched from α_1 to α_{max} for the purpose of reducing time to travel. If θ_2 begins to decrease, which corresponds to the point that the averaged trajectory crosses the

Table 1 Control patterns based upon initial and desired positions.

θ_{2d} / θ_{20}	$[0, \frac{\pi}{2})$	$[\frac{\pi}{2}, \pi)$	$[\pi, \frac{3\pi}{2})$	$[\frac{3\pi}{2}, 2\pi$
$[0, \frac{\pi}{2})$	#1	#2	#3	#4
$[\frac{\pi}{2}, \pi)$	#2′	#1′	#4′	#3′
$[\pi, \frac{3\pi}{2})$	#3	#4	#1	#2
$[\frac{3\pi}{2}, 2\pi$	#4′	#3′	#2′	#1′

1) ′ denotes patterns in which initial and desired positions are reversed.

2) $\dot{\theta}_2(0) = \dot{\theta}_{2d} = 0$ are assumed.

horizontal axis, the amplitude is switched again to α_3 which is supposed to be smaller than α_1. Finally, when θ_2 becomes $\pi/2$, we now enter the last cruising quarter to the target position. The amplitudes in the last quarter are continuously modulated according to law (31) in each oscillation. The above is summarized as pattern #1 below. Control algorithms according to various initial and desired states are summarized in Table 1.

Pattern #1

(1) Set $\alpha = \alpha_1$ and $\omega = \omega_1$. (In Fig. 6 $\alpha_1 = 0.5$ and $w_1 = 4\pi$ are used.)

(2) If $\theta_2(t)$ becomes $\pi/2$, then switch $\alpha = \alpha_{\max}$. (In Fig. 6 $\alpha_{\max} = 0.7$ is used.)

(3) If $\theta_2(t)$ becomes less than its previous value, then switch $\alpha = \alpha_3$, where $\alpha_3 \leq \alpha_1$.

(4) If $\theta_2(t)$ becomes $\pi/2$, then change α sequentially in each step according to (31).

(5) Finally, if $\|\theta_2(t)\ \theta_d\| < \delta$, then switch $\omega = \omega_2$ (In Fig. 6 $\omega_2 = 8\pi$ is used.)

Pattern #2

(1) Set $\alpha = \alpha_1$ and $\omega = \omega_1$.

(2) If $\theta_2(t)$ becomes $\pi/2$, then change α sequentially in each step according to (31).

(3) Finally, if $\|\theta_2(t)\ \theta_d\| < \delta$, then switch $\omega = \omega_2$.

Pattern #3

(1) Set $\alpha = \alpha_1$ and $\omega = \omega_1$.

(2) If $\theta_2(t)$ becomes $\pi/2$, then choose $\alpha_2 < \alpha$

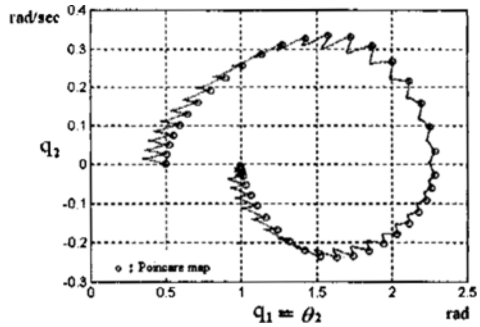


Fig. 6 Positioning from initial angle $\theta_2 = 0.5$ rad to terminal angle $\theta_2 = 1.0$ rad via Pattern #1 ($\alpha_1 = 0.5$, $\alpha_{\max} = 1$, $\alpha_3 = 0$, $\omega_1 = 4\pi$, $\omega_2 = 8\pi$).

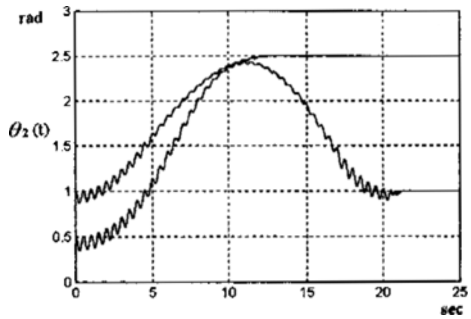


Fig. 7 The motions of the free joint with vibrational controls:

1) Upper plot (1.0, 0) → (2.5, 0)

2) Lower plot (0.5, 0) → (1.0, 0)

$(\pi, 0) \equiv \sqrt{\dfrac{2y_1^2}{n^2(1-\cos^2 y_1)}}$ where $\alpha(\pi, 0)$ is the magnitude of vibrations whose averaged trajectory passes through $(\pi, 0)$.

(3) If $\theta_2(t)$ becomes $3\pi/2$, then switch $\alpha = \alpha_{\max}$.

(4) If $\theta_2(t)$ becomes less than its previous value, then switch $\alpha = \alpha_4$, where $\alpha_4 \leq \alpha_1$.

(5) If $\theta_2(t)$ becomes $3\pi/2$, then change α sequentially in each step according to (31).

(6) Finally, if $\|\theta_2(t)\ \theta_d\| < \delta$, then switch $\omega = \omega_2$.

Pattern #4

(1), (2) are the same as Pattern #3.

(3) If $\theta_2(t)$ becomes $3\pi/2$, then change α sequentially in each step according to (31).

(4) Finally, if $\|\theta_2(t)\ \theta_d\| < \delta$, then switch $\omega = \omega_2$.

Finally, it is noted that the first link needs to be

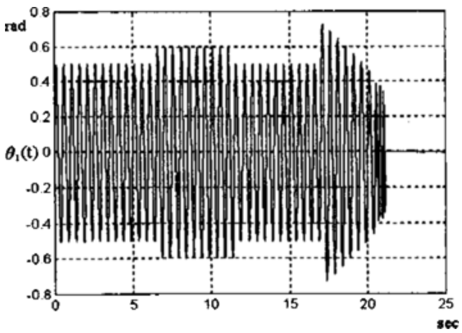


Fig. 8 The oscillating motion of the first joint for the second case of Fig. 7, i.e. (0.5, 0) → (1.0, 0)

stopped at the exact period of input in order to keep it at its desired position. It is also noted that once the free link crosses over its target position, there is no return and it has to go all the way around again. Therefore, just before getting to the target position the input frequency needs to be increased. A precise ending is shown in Fig. 6 at the last stage of control.

5. Conclusions

Open loop vibrational control for underactuated mechanical systems was investigated. An important example of this class is the system with failed actuators. The control strategy to move an unactuated joint in this paper is to utilize the dynamic coupling between the actuated and unactuated parts, which occurs when oscillating the actuated joint. With the vibrations introduced, the whole system becomes time-varying. Hence, the stability analysis of the time-varying system is carried out through the averaging analysis. The averaging method was extended to the system with the derivatives and anti-derivatives of the vibrations introduced. A systematic way of obtaining averaged systems via the generating equations for underactuated systems was developed for the first time. To illustrate the design procedure, a 2R planar manipulator with a free joint was demonstrated. The calculation procedure of the amplitudes and frequencies of vibrations was demonstrated. In the zero gravity outer space the manipulator with a failed joint can be controlled in this approach. The control strategy in this paper provides a viable tool when the conventional control scheme is not applicable and/or actuator failure occurs.

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